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FACULTY OF APPLIED SCIENCE AND ENGINEERING
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By

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THE ADJUSTMENT AND COMPUTATION OF THE GULL LAKE TRIANGULATION

By L. B. STEWART, *Professor of Surveying and Geodesy*

On the establishment of the University of Toronto survey camp at the head of Gull Lake, in the Township of Lutterworth, Ontario, the advantage to be derived from the establishment of triangulation stations near the camp, and their connection with some primary station of the Geodetic Survey of Canada, was at once recognized. The staff in Surveying therefore resolved to carry out a survey with that object at the earliest opportunity.

During a conversation with the Director of the Geodetic Survey in the fall of 1921 he suggested that a good plan would be to lay out a chain of secondary triangulation connecting the primary stations Anson and Somerville, the former of which is situated about five miles west of the camp, and the latter almost due south of it. Points near the camp could then be included in the triangulation scheme, and the lengths of the lines could be made to depend upon that of the line joining those two stations as given by the primary triangulation, thus obviating the necessity for measuring a base.

In the summer of 1923 the opportunity occurred for putting this into effect, the reconnaissance being begun by the writer early in June of that year. On visiting Somerville station, which is situated near the south boundary of the township bearing that name, it was found that Anson station, about twenty-one miles distant, was invisible, owing, as was afterwards found, to the growth of trees on intervening high ground since the angles at those stations were observed. It was therefore decided to dispense with the direct use of that line.

The secondary stations, Coboconk, Dongola, Laxton, Miner's Bay, and Gull Lake, were then established in the positions and giving the figure shown in Fig. 1. The last-named station, being within a quarter of a mile of the survey camp, is conveniently placed to serve for connecting the surveys made in the neighbourhood of the camp with the primary triangulation network of the Dominion.

Two of the stations, Laxton and Dongola, required observing towers 78 and 40 feet in height, respectively, in order that the sight lines to the surrounding stations might clear the neighbouring trees. These were built by a tower-building party of the Geodetic Survey.

The angles were observed with a theodolite having a 7-inch circle read by two opposite verniers to 10″. The method of observation was

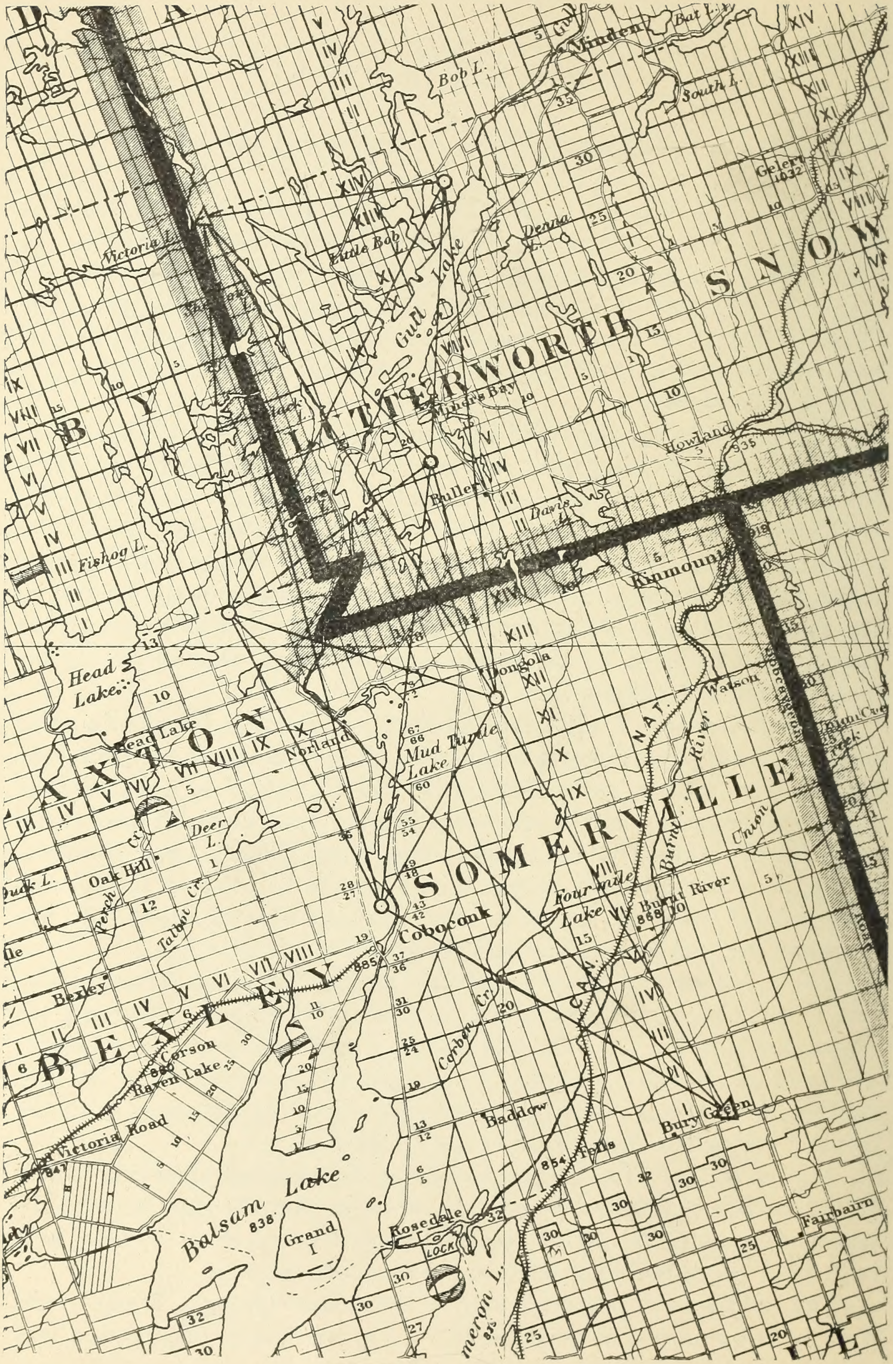

Minden
Bat L.
Bob L.
South L.
Gelert 1032
SNOW
Little Bob L.
Gull Lake
Denna L.
Victoria L.
LUTTERWORTH
Fishers Bay
Buller
Davis L.
Howland 935
Kinmount
B
Fishog L.
Head Lake
Head Lake
LAXTON
Norland
Dongola
Mud Turtle Lake
N.A.T.
Watson
Burnt River
SOMERVILLE
Union Creek
Deer L.
Oak Hill
Duck L.
Perch Cr.
Talbot Cr.
Four-mile Lake
Burnt River 868
Coboconk
885
Bexley
BEXLEY
Corson
Raven Lake 880
Corben Cr.
Victoria Road 841
Baddow
Fells 854
Bury Green
Balsam Lake 838
Grand I.
Rosedale
LOCK
Cameron L.
Fairbairn

FIG. 1

by repetitions, twelve repetitions of each angle being made six of the angle itself with clamp right, and six of the explement of the angle with clamp left. The angles observed are shown in Fig. 2, numbered 1 to 27, their values being as follows:

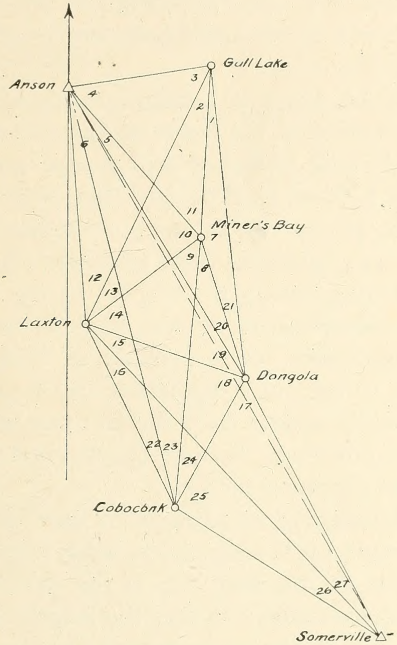


Fig. 2 — Diagram of Triangulation

FIG. 2

Angle	Obs'd value	Angle	Obs'd value
	° ′ ″		° ′ ″
1	8 25 46.0	15	27 24 30.1
2	22 36 20.1	16	17 41 54.8
3	55 55 00.0	17	57 04 38.1
4	56 56 11.8	18	80 11 05.4
5	10 49 40.0	19	40 47 49.6
6	26 20 40.7	20	15 12 19.2
7	161 29 25.8	21	10 04 48.0
8	21 38 01.8	22	12 07 14.8
9	46 41 09.1	23	20 24 24.1
10	85 38 55.5	24	22 10 44.9
11	44 32 25.8	25	93 04 23.5
12	29 58 28.6	26	14 31 14.1
13	27 12 19.7	27	15 19 46.3
14	55 40 44.9		

The ultimate object of a trigonometric survey is to determine the geodetic latitudes and longitudes of the stations, and the azimuths of the lines joining them. The first step towards this end is the adjustment of the angles. This means the determination of the most probable system of corrections to the angles, which will at the same time give corrected values that will exactly fulfil all possible geometrical conditions. The length of any line may then be computed from that of any other, and the same result will be obtained irrespective of the route followed in going from one to the other.

In any triangulation there is a minimum number of angles which will permit of the plotting of the figure or the solution of the triangles, and each additional angle observed will give a geometrical condition that must be exactly fulfilled by the angles concerned. These conditions are also of two sorts, giving rise to angle equations and side equations, respectively, the former resulting from relations among the angles, and the latter from relations among the sides. In the triangulation under discussion there are 10 necessary angles, while 27 were observed; there are therefore 17 conditions, each capable of being expressed by an equation. Of these 10 are angle equations and 6 side equations; there is also a station equation arising from the fact that at one of the stations all the angles were observed, closing the horizon; the sum of their adjusted values must therefore be 360°.

The theory of the method of adjustment used cannot be given here, but it will be illustrated by the numerical work given below.

After the adjustment the next step is the solution of the triangles. In this instance it was necessary to find the lengths of the lines in terms of that of the line joining the primary stations Anson and Somerville, and as that line had not been observed it was necessary to resort to an indirect process. A length was assumed for the line between Dongola and Somerville, the triangles solved, and thence a provisional length found for the line between Anson and Somerville. The ratio of the correct to the provisional length of that line, being the same for all the lines used in determining the latter, served for finding the corrected lengths of those lines.

The final operation is the computation of the geodetic positions and azimuths, and the method used below in making these computations is essentially the same as that of the Geodetic Survey of Canada.

The numerical work involved in carrying out the operations outlined above follow.

The condition equations used in the adjustment of the triangulation are as follows:

Ten angle equations given by the triangles AGM, AML, AGL, LMD, ADL, GDM, MLC, MDC, LSC, and LDS;

Six side equations given by the quadrilaterals AGML, AMDL, GDLM, MDCL, ADCL, and LDSC; and

One station equation given by the angles about station M.
NOTE.—The stations are denoted by their initial letters.

FORMATION OF THE CONDITION EQUATIONS.

SPHERICAL EXCESSES OF TRIANGLES—

A preliminary solution of the triangles is made, assuming $AG=5$ miles.
Formula for spherical excess—

$$e=\frac{a\,b\sin C}{2\,\rho_m\rho_n\sin 1''}=m.ab\sin C$$

$$m=\frac{1}{2\,\rho_m\,\rho_n\sin 1''}$$

		Log	
Triangle AGM—	a	0.69897	
	b	0.77622	
$C=G=78°\ 31'\ 20''$	$\sin C$	9.99123	
	5280^2	7.44526	
	m	$\overline{10}.37204$	
	e	$\overline{1}.28372$	$e=0''.192$
Triangle AML—	a	0.70090	
	b	0.84418	
$C=M=85°\ 38'\ 55''$	$\sin C$	9.99875	
	5280^2	7.44526	
	m	$\overline{10}.37204$	
	e	$\overline{1}.36113$	$e=0''.230$
Triangle AGL—	a	0.69897	
	b	0.91846	
$C=A=94°\ 06'\ 33''$	$\sin C$	9.99888	
	5280^2	7.44526	
	m	$\overline{10}.37204$	
	e	$\overline{1}.43361$	$e=0''.271$
Triangle LMD—	a	0.70090	
	b	0.69924	
$C=M=68°\ 19'\ 11''$	$\sin C$	9.96814	
	5280^2	7.44526	
	m	$\overline{10}.37205$	
	e	$\overline{1}.18559$	$e=0''.153$
Triangle ADL—	a	0.91846	
	b	0.75046	
$C=L=112°\ 51'\ 34''$	$\sin C$	9.96448	
	5280^2	7.44526	
	m	$\overline{10}.37204$	
	e	$\overline{1}.45070$	$e=0''.282$

Triangle *GDM*—	*a*	0.77622	
	b	0.69924	
$C=M=161°\ 29'\ 26''$	sin *C*	9.50169	
	5280^2	7.44526	
	m	$\overline{10}.37204$	
	e	$\bar{2}.79445$	$e=0''.062$
Triangle *MLC*—	*a*	0.70090	
	b	0.83226	
$C=L=100°\ 47'\ 10''$	sin *C*	9.99226	
	5280^2	7.44526	
	m	$\overline{10}.37205$	
	e	$\bar{1}.34273$	$e=0''.220$
Triangle *MDC*—	*a*	0.69924	
	b	0.68896	
$C=D=136°\ 11'\ 14''$	sin *C*	9.84029	
	5280^2	7.44526	
	m	$\overline{10}.37205$	
	e	$\bar{1}.04580$	$e=0''.111$
Triangle *LSC*—	*a*	0.83226	
	b	0.91593	
$C=C=147°\ 46'\ 47''$	sin *C*	9.72687	
	5280^2	7.44526	
	m	$\overline{10}.37205$	
	e	$\bar{1}.29237$	$e=0''.196$
Triangle *LDS*—	*a*	0.75046	
	b	0.99134	
$C=D=137°\ 15'\ 43''$	sin *C*	9.83165	
	5280^2	7.44526	
	m	$\overline{10}.37205$	
	e	$\bar{1}.39076$	$e=0''.246$
Triangle *LDC*—	*a*	0.75046	
	b	0.83226	
$C=L=45°\ 06'\ 25''$	sin *C*	9.85030	
	5280^2	7.44526	
	m	$\overline{10}.37205$	
	e	$\bar{1}.25033$	$e=0''.178$

ANGLE EQUATIONS.

The corrections to the angles are denoted by the corresponding numbers bracketed.

Triangle *AGM*—

° ′ ″
2 = 22 36 20.1
3 = 55 55 00.0
4 = 56 56 11.8
11 = 44 32 25.8

179 59 57.7
180 00 00.2

−2.5

$(2)+(3)+(4)+(11)-2.5=0$

Triangle *AML*—

° ′ ″
5 = 10 49 40.0
6 = 26 20 40.7
10 = 85 38 55.5
12 = 29 58 28.6
13 = 27 12 19.7

180 00 04.5
180 00 00.2

+4.3

$(5)+(6)+(10)+(12)+(13)+4.3=0$

Triangle *AGL*—

° ′ ″
3 = 55 55 00.0
4 = 56 56 11.8
5 = 10 49 40.0
6 = 26 20 40.7
12 = 29 58 28.6

180 00 01.1
180 00 00.3

+0.8

$(3)+(4)+(5)+(6)+(12)+0.8=0$

Triangle *LMD*—

° ′ ″
8 = 21 38 01.8
9 = 46 41 09.1
14 = 55 40 44.9
19 = 40 47 49.6
20 = 15 12 19.2

180 00 04.6
180 00 00.2

+4.4

$(8)+(9)+(14)+(19)+(20)+4.4=0$

Triangle *ADL*—

° ′ ″
6 = 26 20 40.7
12 = 29 58 28.6
13 = 27 12 19.7
14 = 55 40 44.9
19 = 40 47 49.6

180 00 03.5
180 00 00.3

+3.2

$(6)+(12)+(13)+(14)+(19)+3.2=0$

Triangle *GDM*—

° ′ ″
1 = 8 25 46.0
7 = 161 29 25.8
21 = 10 04 48.0

179 59 59.8
180 00 00.1

−0.3

$(1)+(7)+(21)-0,3=0$

Triangle *MLC*—

° ′ ″
9 = 46 41 09.1
14 = 55 40 44.9
15 = 27 24 30.1
16 = 17 41 54.8
22 = 12 07 14.8
23 = 20 24 24.1

179 59 57.8
180 00 00.2

−2.4

$(9)+(14)+(15)+(16)+(22)+(23)-2.4=0$

Triangle *MDC*—

° ′ ″
8 = 21 38 01.8
18 = 80 11 05.4
19 = 40 47 49.6
20 = 15 12 19.2
24 = 22 10 44.9

180 00 00.9
180 00 00.1

+0.8

$(8)+(18)+(19)+(20)+(24)+0.8=0$

Triangle *LSC*—

	°	′	″
16 =	17	41	54.8
22 =	12	07	14.8
23 =	20	24	24.1
24 =	22	10	44.9
25 =	93	04	23.5
26 =	14	31	14.1
	179	59	56.2
	180	00	00.2
			−4.0

$$(16)+(22)+(23)+(24)+(25)+(26)-4.0=0$$

Triangle *LDS*—

	°	′	″
15 =	27	24	30.1
17 =	57	04	38.1
18 =	80	11	05.4
27 =	15	19	46.3
	179	59	59.9
	180	00	00.2
			−0.3

$$(15)+(17)+(18)+(27)-0.3=0$$

SIDE EQUATIONS

Quadrilateral *AGML*. Pole at intersection of diagonals.

$$\frac{\sin 4.\ \sin 2.\ \sin 10.\ \sin 12}{\sin (5+6). \sin 3.\ \sin 11.\ \sin 13}=1$$

	Log. sin	*D*. for 1″		Log. sin	*D*. for 1″
4	9.923279,0	1.37	5+6	9.781191,9	2.77
2	9.584766,7	5.06	3	9.918147,5	1.42
10	9.998746,4	0.16	11	9.845974,0	2.14
12	9.698636,5	3.65	13	9.660090,0	4.10
	9.205428,6			9.205403,4	
	9.205403,4				
	+25.2				

$$5.06\ (2)-1.42\ (3)+1.37\ (4)-2.77\ (5)-2.77\ (6)+0.16\ (10)-2.14\ (11)+3.65\ (12)-4.10\ (13)+25.2=0.$$

Quadrilateral *AMDL*. Pole at *M*.

$$\frac{\sin 20.\ \sin (5+6),\ \sin 14}{\sin 5.\ \sin (12+13)\ \sin (19+20)}=1$$

	Log. sin	D. for 1″		Log. sin	D. for 1
20	9.418763,5	7.74	5	9.273828,6	11.01
5+6	9.781191,9	2.77	12+13	9.924474,8	1.36
14	9.916923,8	1.43	19+20	9.918586,7	1.42
	9.116879,2			9.116890,1	
	9.116890,1				
	−10.9				

−8.24 (5)+2.77 (6)−1.36 (12)−1.36 (13)+1.43 (14)−1.42 (19)+6.32 (20)−10.9 =0.

Quadrilateral *GDLM*. Pole at *M*.

$$\frac{\sin 1 . \sin (19+20) . \sin 13}{\sin 2 . \sin 21 . \sin 14}=1$$

	Log. sin	D. for 1″		Log. sin	D. for 1″
1	9.166108,5	14.21	2	9.584766,7	5.06
19+20	9.918586,7	1.42	21	9.243095,3	11.85
13	9.660090,0	4.10	14	9.916923,8	1.43
	8.744785,2			8.744785,8	
	8.744785,8				
	−0.6				

14.21 (1)−5.06 (2)+4.10 (13)−1.43 (14)+1.42 (19)+1.42 (20)−11.85 (21)−0.6=0

Quadrilateral *MDCL*. Pole at intersection of diagonals.

$$\frac{\sin 8 .\ \sin 18 .\ \sin (22+23) .\ \sin 14}{\sin 9 .\ \sin (19+20)\ \sin 24 .\ \sin (15+16)}=1$$

	Log. sin	D. for 1″		Log. sin	D. for 1″
8	9.566642,0	5.31	9	9.861894,8	1.99
18	9.993596,1	0.36	19+20	9.918586,7	1.42
22+23	9.730543,2	3.30	24	9.576921,2	5.16
14	9.916923,8	1.43	15+16	9.850293,9	2.10
	9.207705,1			9.207696,6	
	9.207696,6				
	+8.5				

5.31 (8)−1.99 (9)+1.43 (14)−2.10 (15)−2.10 (16)+0.36 (18)−1.42 (19) −1.42 (20)+3.30 (22)+3.30 (23)−5.16 (24)+8.5=0.

Quadrilateral *ADCL*. Pole at *A*.

$$\frac{\sin 19 .\ \sin (12+\ .\ .\ 16) .\ \sin (23+24)}{\sin (12+13+14) . \sin 22 .\ \sin (18+19)}=1$$

	Log. sin	*D.* for 1″		Log. sin	*D.* for 1″
19	9.815167,4	2.44	12+13+14	9.964477,5	−0.89
12+ . . 16	9.574210,2	−5.20	22	9.322163,7	9.80
23+24	9.830392,3	2.29	18+19	9.933147,8	−1.26
	9.219769,9			9.219789,0	
	9.219789,0				
	−19.1				

$$-4.31\ (12)-4.31\ (13)-4.31\ (14)-5.20\ (15)-5.20\ (16)+1.26\ (18)+3.70\ (19)-9.80\ (22)+2.29\ (23)+2.29\ (24)-19.1=0.$$

Quadrilateral *LDSC*. Pole at *L*.

$$\frac{\sin 18 .\ \sin (22+\dots 25) .\ \sin 27}{\sin (22+23+24) .\ \sin 26 .\ \sin (17+18)}=1$$

	Log. sin	*D.* for 1″		Log. sin	*D.* for 1″
18	9.993596,1	0.36	22+ . . 24	9.911798,9	1.49
22+ . . 25	9.726869,3	−3.35	26	9.399202,4	8.13
27	9.422212,4	7.68	17+18	9.831643,2	−2.28
	9.142677,8			9.142644,5	
	9.142644,5				
	+33.3				

$$2.28\ (17)+2.64\ (18)-4.84\ (22)-4.84\ (23)-4.84\ (24)-3.35\ (25)-8.13\ (26)+7.68\ (27)+33.3=0.$$

Station equation.

	°	′	″
7=	161	29	25.8
8=	21	38	01.8
9=	46	41	09.1
10=	85	38	55.5
11=	44	32	25.8
	359	59	58.0
	360	00	00.0
			−2.0

$$(7)+(8)+(9)+(10)+(11)-2.0=0.$$

The solution of the normal equations gives the following values for the correlates

$k_1 = -0.1746$
$k_2 = -2.5426$
$k_3 = +1.2382$
$k_4 = -4.0162$
$k_5 = +1.4574$
$k_6 = -0.5124$
$k_7 = +2.0517$
$k_8 = +1.1178$
$k_9 = -1.2179$
$k_{10} = +0.1910$
$k_{11} = +1.8747$
$k_{12} = -0.4297$
$k_{13} = +0.2808$
$k_{14} = -0.0159$
$k_{15} = -0.0397$
$k_{16} = +0.1958$
$k_{17} = -0.2327$

These values, substituted in the correlate equations, give the following values for the corrections, and thence the corrected values of the angles:

Corrections	Corrected angles		
	°	′	″
(1) = −0.7383	1 = 8	25	45.26
(2) = −2.2684	2 = 22	36	17.83
(3) = +1.6738	3 = 55	55	01.67
(4) = +0.4748	4 = 56	56	12.27
(5) = −2.4279	5 = 10	9	37.57
(6) = +2.1211	6 = 26	20	42.82
(7) = +1.3623	7 = 161	29	27.16
(8) = −1.2345	8 = 21	38	00.57
(9) = −0.0108	9 = 46	41	09.09
(10) = −0.7367	10 = 85	38	54.76
(11) = +2.6197	11 = 44	32	28.42
(12) = −2.6412	12 = 29	58	25.96
(13) = −0.6144	13 = 27	12	19.09
(14) = −0.9836	14 = 55	40	43.92
(15) = +1.3079	15 = 27	24	31.41
(16) = −0.1010	16 = 17	41	54.70
(17) = −0.3396	17 = 57	04	37.76
(18) = +0.9269	18 = 80	11	06.33
(19) = −1.0814	19 = 40	47	48.52
(20) = −1.0899	20 = 15	12	18.11
(21) = −0.3240	21 = 10	04	47.68
(22) = −0.0897	22 = 12	07	14.71
(23) = +2.2775	23 = 20	24	26.38
(24) = +1.6795	24 = 22	10	46.58
(25) = −0.4384	25 = 93	04	23.06
(26) = +0.6740	26 = 14	31	14.77
(27) = −1.5961	27 = 15	19	44.70
Mean = 1.1790			

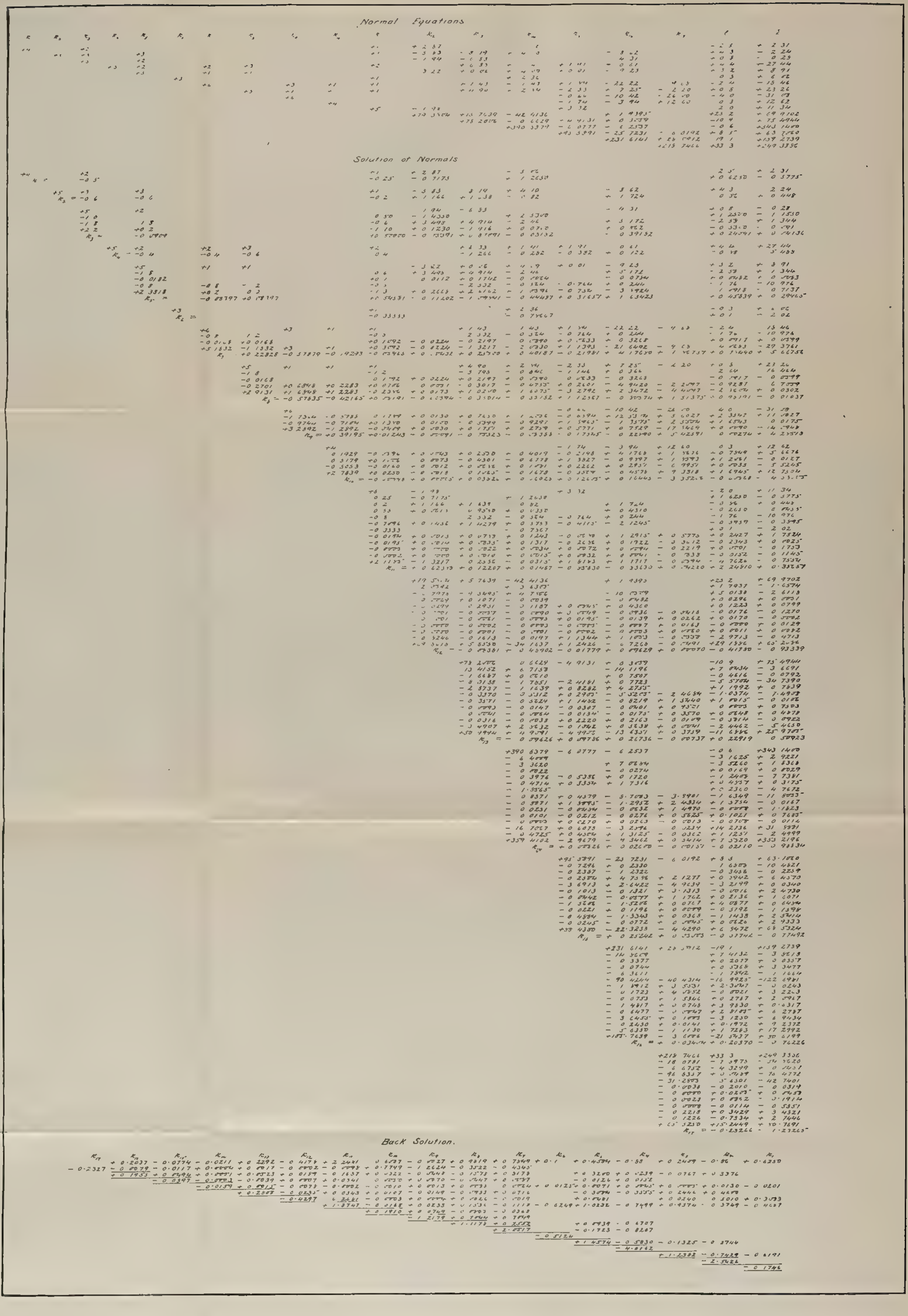
THE CONDITION EQUATIONS ARE THEN AS FOLLOWS:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	
	+1	+1	+1							+1																	− 2 5−0
				+1	+1				+1		+1	+1															+ 4.3=0
		+1	+1 .	+1	+1						+1																+ 0 8=0
							+1	+1					+1					+1	+1								+ 4 4=0
					+1						+1	+1	+1					+1									+ 3 2=0
+1						+1														+1							− 0 3=0
								+1					+1	+1	+1						+1	+1					− 2 4=0
							+1										+1	+1	+1				+1				+ 0 8=0
															+1						+1	+1	+1	+1	+1		− 4 0=0
														+1		+1	+1									+1	− 0 3−0
						+1	+1	+1	+1	+1																	− 2 0=0
	+5 06	+1.42	+1.37	−2.77	−2.77				+0.16	−2.14	+3.65	−4.10															+25 2=0
				−8 24	+2.77						−1 36	−1.36	+1.43					−1.42	+6 32								−10 9=0
+14.21	−5.06											+4.10	−1.43					+1.42	+1 42	−11.85							− 0 6=0
							+5.31	−1.99					+1.43	−2.10	−2.10		+0.36	−1 42	−1 42		+3.30	+3 30	−5.16				+ 8 5=0
											−4 31	−4 31	−4 31	−5.20	−5.20		+1 26	+3 70			−9 80	+2.29	+2.29				−19 1−0
																+2.28	+2.64				−4.84	−4.84	−4.84	−3.35	−8 13	+7 68	+33 3−0

These give the correlate equations, as follows:

	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}	k_{16}	k_{17}
(1) =						+1								+14.01			
(2) =	+1											+5.06		− 5.06			
(3) =	+1		+1									−1 42					
(4) −	+1		+1									+1.37					
(5) −		+1	+1									−2 77	−8.24				
(6) =		+1	+1		+1							−2.77	+2.77				
(7) =						+1					+1						
(8) =				+1				+1			+1				+5.31		
(9) −				+1			+1				+1				−1.99		
(10) −		+1									+1	+0 16					
(11) =	+1										+1	−2 14					
(12) =		+1	+1		+1							+3.65	−1 36			−4 31	
(13) =		+1			+1							−4 10	−1.36	+4.10		−4 31	
(14) =				+1	+1		+1						+1 43	−1.43	+1.43	−4 31	
(15) =							+1			+1					−2.10	−5 20	
(16) =							+1		+1						−2.10	−5.20	
(17) =										+1							+2.28
(18) =								+1		+1					+0.36	+1.26	+2.64
(19) =				+1	+1			+1					−1 42	+1.42	−1.42	+3.70	
(20) −				+1				+1					+6.32	+1.42	−1.42		
(21) =						+1								−11 85			
(22) =							+1		+1						+3 30	−9 80	−4.84
(23) =							+1		+1						+3.30	+2 29	−4.84
(24) =								+1	+1						−5.16	+2 29	−4.84
(25) =									+1								−3.35
(26) =									+1								−8.13
(27) =										+1							+7.68

The values of the corrections given by these equations are now substituted in the condition equations, giving the normal equations, as below.


Normal Equations
Solution of Normals
Back Solution.

SOLUTION OF TRIANGLES.

Log. of assumed length of line $DS = 4.1979876$.

Triangle DSC—

Sph. angles	Pl. angles	Log.
° ′ ″	″	DS.........4.1979876
D=57 04 37.76	37.66	sin C.......9.9993750
S=29 50 59.47	59.38	
C=93 04 23.06	22.96	4.1986126
		sin D.......9.9239705
180 00 00.29		sin S.......9.6969924
		CS.........4.1225831
		CD.........3.8956050

Triangle LDC—

° ′ ″	″	CD.........3.8956050
L=45 06 26.11	26.08	sin L.......9.8502964
D=80 11 06.33	06.29	
C=54 42 27.67	27.63	4.0453086
		sin D.......9.9935965
180 00 00.11		sin C.......9.9118046
		LC.........4.0389051
		LD.........3.9571132

Triangle MDL—

° ′ ″	″	LD.........3.9571132
M=68 19 09.66	09.59	sin M.......9.9681360
D=56 00 06.63	06.56	
L=55 40 43.92	43.85	3.9889772
		sin D.......9.9185835
180 00 00.21		sin L.......9.9169223
		LM.........3.9075607
		MD.........3.9058995

Triangle AML—

° ′ ″	″	LM.........3.9075607
A=37 10 20.39	20.33	sin A.......9.7811909
M=85 38 54.76	54.69	
L=57 10 45.05	44.98	4.1263698
		sin M.......9.9987463
180 00 00.20		sin L.......9.9244703
		AL.........4.1251161
		AM.........4.0508401

Computation of LS—

Formulæ—$\cos\theta = \frac{\sqrt{ab}}{a+b}\, 2 \cos \tfrac{1}{2} C \quad c = (a+b) \sin\theta.$

Triangle *LDS*—

	Log
$LD = 9059.6875$	3.9571132
$DS = 15775.6618$	4.1979876
$LD.DS$	8.1551008
$\sqrt{LD.DS}$	4.0775504
2	0.3010300
$\cos \frac{1}{2} LDS$	9.5615440
$LD + DS = 24835.3493$	4.3950703
	3.9401244
$\cos\theta$ $(\theta = 69^\circ\ 27'\ 50''.46)$	9.5450541
$\sin\theta$	9.9714856
$LD + DS$	4.3950703
LS	4.3665559

Triangle *LCS*—

$LC = 10937.1738$	4.0389051
$CS = 13261.2080$	4.1225831
$LC.CS$	8.1614882
$\sqrt{LC.CS}$	4.0807441
2	0.3010300
$\cos \frac{1}{2} LCS$	9.4432256
$LC + CS = 24198.3818$	4.3837863
	3.8249997
$\cos\theta$ $(\theta = 73^\circ\ 58'\ 00''.66)$	9.4412134
$\sin\theta$	9.9827695
$LC + CS$	4.3837863
LS	4.3665558

Computation of *AD*—

Triangle *AMD*—

	Log.
$AM = 11241.9096$	4.0508401
$MD = 8051.9204$	3.9058995
$AM.MD$	7.9567396
$\sqrt{AM.MD}$	3.9783698
2	0.3010300
$\cos \frac{1}{2} AMD$	9.3526149
$AM + MD = 19293.8300$	4.2854185
	3.6320147
$\cos\theta$ $(\theta = 77^\circ\ 09'\ 58''.18)$	9.3465962
$\sin\theta$	9.9890128
$AM + MD$	4.2854185
AD	4.2744313

Triangle ALD—

$AL = 13338.7791$	4.1251161
$LD = 9059.6875$	3.9571132
$AL.LD$	8.0822293
$\sqrt{AL.LD}$	4.0411146
2	0.3010300
cos ½ ALD	9.7427014
$AL + LD = 22398.4666$	4.3502183
	4.0848460
cos θ ($\theta = 57° 07' 35''.71$)	9.7346277
sin θ	9.9242130
$AL + LD$	4.3502183
AD	4.2744313

Computation of AS—

Triangle ADS—

$AD = 18811.8398$	4.2744313
$DS = 15775.6618$	4.1979876
$AD.DS$	8.4724189
$\sqrt{AD.DS}$	4.2362094
2	0.3010300
cos ½ ADS	8.2288397
$AD + DS = 34587.5016$	4.5389192
	2.7660791
cos θ ($\theta = 89° 01' 59''.79$)	8.2271599
sin θ	9.9999382
$AD + DS$	4.5389192
AS	4.5388574

Triangle ALS—

$AL = 13338.7791$	4.1251161
$LS = 23257.1176$	4.3665559
$AL.LS$	8.4916720
$\sqrt{AL.LS}$	4.2458360
2	0.3010300
cos ½ ALS	9.5312643
$AL + LS = 36595.8967$	4.5634324
	4.0781303
cos θ ($\theta = 70° 54' 22''.88$)	9.5146979
sin θ	9.9754250
$AL + LS$	4.5634324
AS	4.5388574

AS (by Primary Triang'n)	4.5272949
AS (provisional)	4.5388574
Ratio	$\bar{1}.9884375$

Logarithms of corrected lengths of lines—

Line	Log.	Line	Log.
CS	4.1110206	*MD*	3.8943370
CD	3.8840425	*AL*	4.1135536
LC	4.0273426	*AM*	4.0392776
LD	3.9455507	*DS*	4.1864251
LM	3.8959982	*LS*	4.3549934

Triangle *AGM*—

	Sph. angles	Pl. angles	Log.
	° ′ ″	″	
A =	56 56 12.27	12.21	*AM*......4.0392776
G =	78 31 19.50	19.43	sin *G*.....9.9912267
M =	44 32 28.42	28.36	4.0480509
	180 00 00.19		sin *A*.....9.9232795
			sin *M*....9.8459795
			GM......3.9713304
			AG.......3.8940304

Triangle *ALS*—

$LS = 23257.1176$
$AL = 13338.7791$

$LS + AL = 36595.8967$ 4.5634324

$LS - AL = 9918.3385$ 3.9964389
tan ½ $(A+S)$ (=19° 51′ 59″.895) 9.5579118

3.5543507
tan ½ $(A-S)$ (= 5° 35′ 35″.426) 8.9909183

	°	′	″
A =	25	27	35.321
S =	14	16	24.469
L =	140	16	00.21
	180	00	00.000

COMPUTATION OF GEODETIC POSITIONS.

Formulæ used in the computation—

$$x = \frac{s \sin \alpha_1}{N_1} \qquad y = \frac{s \cos \alpha_1}{N_1}$$

$$\Delta\phi^1 = \frac{y}{\sin 1''} + \frac{y^3 \tan^2 \alpha_1}{3 \sin 1''} - \frac{x^2 \tan \phi^1}{2 \sin 1''}$$

$\phi^1 = \phi_1 +$ 1st two terms.

$$\Delta\phi = \Delta\phi^1 \frac{N_1}{R_m}$$

$$\Delta L = \frac{1}{\sin 1''}\, x \sec \phi^1_2$$

$$\Delta\alpha = \Delta L \sin \phi_m \sec \tfrac{1}{2} \Delta\phi \qquad \alpha_2 = \alpha_1 + 180^\circ + \Delta\alpha.$$

In these formulæ—

ϕ_1 = initial latitude
α_1 = initial azimuth
s = length of line
$\Delta\phi^1$ = diff. of latitude on a sphere of radius N_1
$\Delta\phi$ = diff. of latitude on the spheroid
ΔL = diff. of longitude
$\Delta\alpha$ = convergence of meridians
$\phi^1_2 = \phi_1 + \Delta\phi^1$
$\phi_m = \phi_1 + \frac{1}{2}\Delta\phi$
N_1 = length of normal for latitude ϕ_1
R_m = radius of curvature of meridian for latitude ϕ_m
α_2 = reverse azimuth

GEODETIC POSITIONS.

Line—Anson to Gull Lake.

		° ′ ″
α Anson to Somerville	=	150 09 20.28
$\angle$ Somerville to Gull Lake	= −	68 38 57.17
α Anson to Gull Lake	=	81 30 23.11

° ′ ″		° ′ ″
ϕ_1 = 44 52 35.389	α_1 =	81 30 23.11
$\Delta\phi$ = +37.336	$180^\circ + \Delta\alpha$ =	180 04 09.16
ϕ_2 = 44 53 12.725	α_2 =	261 34 22.27
L_1 = 78 52 05.935		
$-\Delta L$ = −5 53.096		
L_2 = 78 46 12.839		

		Log.
s		3.8940304
N_1		6.8054315
s/N_1		$\bar{3}.0885989$
sin α_1		9.9952105
cos α_1		9.1693763
x		$\bar{3}.0838094$
y		$\bar{4}.2579752$
$^1/\sin 1''$		5.3144251
1st term	37″.3594	1.5724003
y^3		$\overline{12}.7739256$
$\tan^2 \alpha_1$		1.6516684
$^1/3 \sin 1''$		4.83730
2nd term	0	$\bar{5}.26289$
x^2		$\bar{6}.1676188$
tan ϕ^1	$\phi^1 = 44^\circ\ 53'\ 12''.748$	9.9983272
½ sin 1″		5.0133951
−3rd term	−0″.1511	$\bar{1}.1793411$
$\Delta\phi^1$	37″.2083	1.5706402
N_1		6.8054315
R_m		6.8039483
		8.3760717
$\Delta\phi$	37″.3356	1.5721234
x		$\bar{3}.0838094$
$^1/\sin 1''$		5.3144251
sec $\phi^1{}_2$	$\phi^1{}_2 = 44^\circ\ 53'\ 12''.597$	0.1496589
ΔL	353″.0965	2.5478934
sin ϕ_m	$\phi_m = 44^\circ\ 52'\ 54''.057$	9.8485863
sec ½ $\Delta\phi$		0
$\Delta\alpha$	249″.1608	2.3964797

Line—Anson to Miner's Bay.

		° ′ ″
α Anson to Somerville	=	150 09 20.28
$\angle$ Somerville to Miner's Bay	= −	11 42 44.90
α Anson to Miner's Bay	=	138 26 35.38

	° ′ ″		° ′ ″
$\phi_1 =$	44 52 35.389	$\alpha_1 =$	138 26 35.38
$\Delta\phi =$	−4 25.491	$180° + \Delta\alpha =$	180 03 52.98
$\phi_2 =$	44 48 09.898	$\alpha_2 =$	318 30 28.36

$L_1 =$	78 52 05.935
$-\Delta L =$	−5 30.407
$L_2 =$	78 46 35.528

		Log.
s		4.0392776
N_1		6.8054315
s/N_1		$\bar{3}$.2338461
sin α_1		9.8217510
cos α_1		9.8740746n
x		$\bar{3}$.0555971
y		$\bar{3}$.1079207n
$^1/\sin 1''$		5.3144251
1st term	−264″.4513	2.4223458n
y^3		$\bar{9}$.32376n
$\tan^2\alpha_1$		9.89535
$1/3 \sin 1''$		4.83730
2nd term	−0″.00011	$\bar{4}$.05641n
x^2		$\bar{6}$.1111942
tan ϕ^1	$\phi^1 = 44°\ 48'\ 10''.938$	9.9970141
½ sin 1″		5.0133951
−3rd term	−0″.13231	$\bar{1}$.1216034
$\Delta\phi^1$	−264″.58376	2.4225632n
N_1		6.8054315
R_m		6.8039450
		9.2279947n
$\Delta\phi$	−265″.49092	2.4240497n
x		$\bar{3}$.0555971
1/sin 1″		5.3144251
sec ϕ^1_2	$\phi^1_2 = 44°\ 48'\ 10''.805$	0.1490269
ΔL	330″.4069	2.5190491
sin ϕ_m	$\phi_m = 44°\ 50'\ 22''.644$	9.8482660
sec ½ $\Delta\phi$		0.0000001
$\Delta\alpha$	232″.9782	2.3673152

Line—Anson to Laxton.

	° ′ ″
α Anson to Somerville	$=150\ 09\ 20.28$
$\angle$ Somerville to Laxton	$=\ 25\ 27\ 35.49$
α Anson to Laxton	$=175\ 36\ 55.77$

	° ′ ″		° ′ ″
$\phi_1=$	$44\ 52\ 35.389$	$\alpha_1=$	$175\ 36\ 55.77$
$\Delta\phi=$	$-6\ 59.532$	$180°+\Delta\alpha=$	$180\ 00\ 31.82$
$\phi_2=$	$44\ 45\ 35.857$	$\alpha_2=$	$355\ 37\ 27.59$

$L_1=$	$78\ 52\ 05.935$
$-\Delta L=$	-45.147
$L_2=$	$78\ 51\ 20.788$

		Log.
s		4.1135536
N_1		6.8054315
s/N_1		$\bar{3}.3081221$
$\sin \alpha_1$		8.8833743
$\cos \alpha_1$		$9.9987271n$
x		$\bar{4}.1914964$
y		$\bar{3}.3068492n$
$1/\sin 1''$		5.3144251
1st term	$-418''.0943$	$2.6212743n$
y^3		$\bar{9}.9205476n$
$\tan {}^2\alpha_1$		7.7692944
$1/3 \sin 1''$		4.83730
2nd term		$\bar{6}.52714n$
x^2		$\bar{8}.3829928$
$\tan \phi^1$	$\phi^1=44°\ 45'\ 37''.295$	9.9963671
$\frac{1}{2} \sin 1''$		5.0133951
$-$3rd term	$-0''.00247$	$\bar{3}.3927550$
$\Delta\phi^1$	$-418''.0968$	$2.6212769n$
N_1		6.8054315
R_m		6.8039434
		$9.4267084n$
$\Delta\phi$	$-419''.5319$	$2.6227650n$
x		$\bar{4}.1914964$
$1/\sin 1''$		5.3144251
$\sec \phi^1_2$	$\phi^1_2=44°\ 45'\ 37''.292$	0.1487061
ΔL	$45''.14686$	1.6546276
$\sin \phi_m$	$\phi_m=44°\ 49'\ 05''.624$	9.8481028
$\sec \frac{1}{2}\Delta\phi$		0.0000002
$\Delta\alpha$	$31''.8222$	1.5027306

Line—Laxton to Coboconk.

		°	′	″
α Laxton to Anson	=	355	37	27.59
∠ Anson to Coboconk	= −	202	02	04.92
α Laxton to Coboconk	=	153	35	22.67

	°	′	″
$\phi_1 =$	44	45	35.857
$\Delta\phi =$		−5	09.058
$\phi_2 =$	44	40	26.799

$\alpha_1 =$	153	35	22.67
$180 + \Delta\alpha =$	180	02	31.32
$\alpha_2 =$	333	37	53.99

$L_1 =$	78	51	20.788
$-\Delta L =$		−3	35.061
$L_2 =$	78	47	45.727

		Log.
s		4.0273426
N_1		6.8054285
s/N_1		$\bar{3}.2219141$
$\sin \alpha_1$		9.6481621
$\cos \alpha_1$		9.9521293n
x		$\bar{4}.8700762$
y		$\bar{3}.1740434n$
$1/\sin 1''$		5.3144251
1st term	−307″.9417	2.4884685n
y^3		$\bar{9}.5221302n$
$\tan^2\alpha_1$		9.3920656
$1/3 \sin 1''$		4.83730
2nd term	0″.000056	$\bar{5}.75150n$
x^2		$\bar{7}.7401524$
$\tan \phi^1$	$\phi^1 = 44°\ 40'\ 27''.915$	9.9950642
½ $\sin 1''$		5.0133951
−3rd term	−0″.05605	$\bar{2}.7486117$
$\Delta\phi^1$	−307″.9979	2.4885477n
N_1		6.8054285
R_m		6.8039356
		9.2939762n
$\Delta\phi$	−309″.0584	2.4900406n
x		$\bar{4}.8700762$
$1/\sin 1''$		5.3144251
$\sec \phi^1_2$	$\phi^1_2 = 44°\ 40'\ 27''.859$	0.1480610
ΔL	215″.0613	2.3325623
$\sin \phi_m$	$\phi = 44°\ 43'\ 01''.328$	9.8473295
$\sec$ ½ $\Delta\phi$		0.0000001
$\Delta\alpha$	151″.3185	2.1798919

Line—Coboconk to Somerville.

		° ′ ″
α Coboconk to Laxton	=	333 37 53.99
$\angle$ Laxton to Somerville	=	−212 13 09.27
α Coboconk to Somerville	=	121 24 44.72

	° ′ ″		
$\phi_1 =$	44 40 26.799	$\alpha_1 =$	121 24 44.72
$\Delta\phi =$	−3 38.334	$180 + \Delta\alpha =$	180 05 51.21
$\phi_2 =$	44 36 48.465	$\alpha_2 =$	301 30 35.93
$L_1 =$	78 47 45.727		
$-\Delta L =$	−8 19.802		
$L_2 =$	78 39 25.925		

		Log.
s		4.1110206
N_1		6.8054263
s/N_1		3.3055943
sin α_1		9.9311719
cos α_1		9.7170000n
x		3.2367662
y		3.0225943n
1/sin 1″		5.3144251
1st term	−217″.2798	2.3370194n
y^3		9.0677829n
$\tan^2\alpha_1$		0.4283438
1/3 sin 1″		4.83730
2nd term	−0″.00021	4.33343n
x^2		6.4735324
tan ϕ^1	$\phi^1 = 44°\ 36'\ 49''.519$	9.9941445
½ sin 1″		5.0133951
−3rd term	−0″.30274	1.4810720
$\Delta\phi^1$	−217″.5827	2.3376244n
N_1		6.8054263
R_m		6.8039299
		9.1430507n
$\Delta\phi$	−218″.3337	2.3391208n
x		3.2367662
1/sin 1″		5.3144251
sec ϕ^1_2	$\phi^1_2 = 44°\ 36'\ 49''.216$	0.1476064
ΔL	499″.8016	2.6987977
sin ϕ_m	$\phi_m = 44°\ 38'\ 37''.632$	9.8467681
sec ½ $\Delta\phi$		0.0000001
$\Delta\alpha$	351″.2092	2.5455659

Line—Miner's Bay to Dongola.

		′ ″
α Miner's Bay to Anson	=	318 30 28.36
∠ Anson to Dongola	=	−153 58 04.42
α Miner's Bay to Dongola	=	164 32 23.94

	° ′ ″		° ′ ″
ϕ_1 =	44 48 09.898	α_1 =	164 32 23.94
$\Delta\phi$ =	−4 04.814	180 + $\Delta\alpha$ =	180 01 06.89
ϕ_2 =	44 44 05.084	α_2 =	344 33 30.83

L_1 =	78 46 35.528
$-\Delta L$ =	−1 34.984
L_2 =	78 45 00.544

		Log.
s		3.8943370
N_1		6.8054296
s/N_1		$\bar{3}$.0889074
sin α_1		9.4258045
cos α_1		9.9839945n
x		$\bar{4}$.5147119
y		$\bar{3}$.0729019n
1/sin 1″		5.3144251
1st term	−243″.96472	2.3873270n
y^3		$\bar{9}$.2187057n
$\tan^2\alpha_1$		8.8836200
1/3 sin 1″		4.83730
2nd term	−0.	$\bar{6}$.93963n
x^2		$\bar{7}$.0294238
tan ϕ^1	ϕ^1 = 44° 44′ 05″.933	9.9959823
½ sin 1″		5.0133951
−3rd term	0″.01093	$\bar{2}$.0388012
$\Delta\phi^1$	−243″.97566	2.3873465n
N_1		6.8054296
R_m		6.8039396
		9.1927761n
$\Delta\phi$	−244″.81412	2.3888365n
x		$\bar{4}$.5147119
1/sin 1″		5.3144251
sec ϕ^1_2	ϕ^1_2 = 44° 44′ 05″.922	0.1485154
ΔL	94″.98442	1.9776524
sin ϕ_m	ϕ_m = 44° 46′ 07″.491	9.8477251
sec ½ $\Delta\phi$		0.0000001
$\Delta\alpha$	66″.89252	1.8253776

Line—Dongola to Somerville.

		° ′ ″
α Dongola to Miner's Bay	=	344 33 30.83
$\angle$ Miner's Bay to Somerville	=	−193 15 50.72
α Dongola to Somerville	=	151 17 40.11

	° ′ ″		° ′ ″
$\phi_1 =$	44 44 05.084	$\alpha_1 =$	151 17 40.11
$\Delta\phi$	−7 16.619	$180 + \Delta\alpha =$	180 03 55.26
$\phi_2 =$	44 36 48.465	$\alpha_2 =$	331 21 35.37

$L_1 =$	78 45 00.544
$-\Delta L =$	−5 34.619
$L_2 =$	78 39 25.925

		Log.
s		4.1864251
N_1		6.8054279
s/N_1		$\bar{3}$.3809972
sin α_1		9.6815199
cos α_1		9.9430490*n*
x		$\bar{3}$.0625171
y		$\bar{3}$.3240462*n*
1/sin 1″		5.3144251
1st term	−434″.9820	2.6384713*n*
y^3		$\bar{9}$.9721386*n*
$\tan^2\alpha_1$		9.4769418
1/3 sin 1″		4.83730
2nd term	−0.″00019	$\bar{4}$.28638*n*
x^2		$\bar{6}$.1250342
tan ϕ^1	$\phi^1 = 44°\ 36'\ 50''.102$	9.9941469
½ sin 1″		5.0133951
−3rd term	−0″.135699	$\bar{1}$.1325762
$\Delta\phi^1$	−435″.1179	2.6386066*n*
N_1		6.8054279
R_m		6.8039323
		9.4440345*n*
$\Delta\phi$	−436″.61859	2.6401022*n*
x		$\bar{3}$.0625171
1/ sin 1″		5.3144251
sec $\phi^1{}_2$	$\phi^1{}_2 = 44°\ 36'\ 49''.966$	0.1476079
ΔL	334″.61862	2.5245501
sin ϕ_m	$\phi_m = 44°\ 40'\ 26''.775$	9.8470006
sec ½ $\Delta\phi$		0.0000002
$\Delta\alpha$	235″.26151	2.3715509

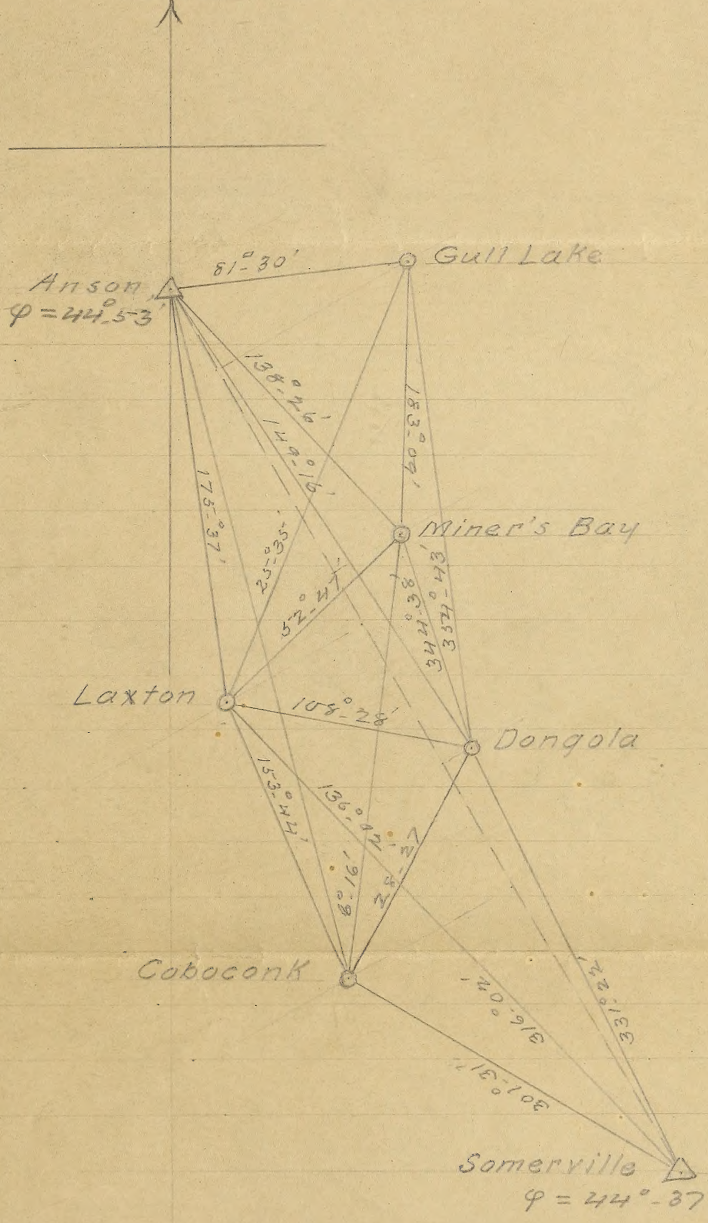
The geographical positions of the triangulation stations may then be tabulated as follows:

Sta.	Lat.	Long.
	° ′	° ′ ′
Anson (G.S.C.)	44 52 35.389	78 52 05.935
Gull Lake	44 53 12.725	78 46 12.839
Miner's Bay	44 48 09.898	78 46 35.528
Laxton	44 45 35.857	78 51 20.788
Dongola	44 44 05.084	78 45 00.544
Coboconk	44 40 26.799	78 47 45.727
Somerville (G.S.C.)	44 36 48.465	78 39 25.926

The agreement between the values for the latitude and longitude of Somerville, given on pp. 22 and 24, serves as a final check on all the numerical work.

DEPARTMENT OF THE INTERIOR

GEODETIC SURVEY OF CANADA
NOEL J. OGILVIE, SUPERINTENDENT



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